JIVE Uniboard Correlator Memo 6: The Delay Model Revisited

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1 Introduction

On 7 June 2012 the JUC team met with Sergei Pogrebrenko to discuss the delay model implementation for the Uniboard correlators. This report summarises my understanding of this and my own previous discussions with Sergei. Note that the implications of space VLBI (i.e., Radio ASTRON) are not discussed here; I will write a separate note for that.

A schematic diagram for the JUC delay implementation is shown in Drawing 1.

In summary:

- The delay model should be calculated for every FFT segment (i.e., blocks of 1024 samples) in units of samples
- The delay is split into
  - integer delay, used to shift the whole segment
  - fractional delay, used to generate phase correction vector after polyphase filter-bank (PFB).
- The phase model is calculated for each sample

It had previously been proposed to calculate the delay and phase by means of quadratic polynomials, the coefficients of which would be refreshed every 1/32 seconds. Calculations then showed that for likely delay values this was overkill: the quadratic term never contributed anything but zero to the delay.

2 Delay accuracy

2.1 The current proposal

As shown in JUC Memo 2, the maximum quadratic coefficient (half the delay acceleration) for terrestrial VLBI is $5 \times 10^{-11} s/s^2$. The current proposal is for a 32-bit fixed point delay coefficient with 8 bits after the binary point, so that the time resolution, t_{res}, is $\frac{1}{32 \times 10^6} \cdot 2^{-8} = 1.2 \times 10^{-10}$ s, and for delay and delay-rate registers but no coefficient for the quadratic term in the delay model. The fractional part of the delay is used as an index into a lookup table for phase correction after the polyphase filter bank; at present the first 4 bits of the fractional part are used for this.

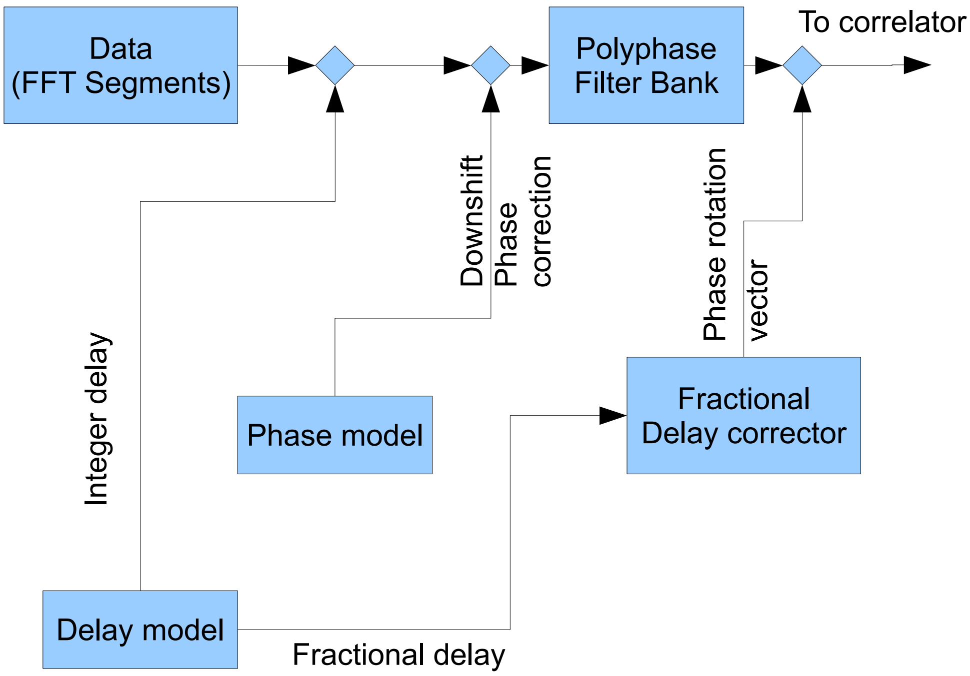

Figure 1: Delay errors with 28-bit shifted linear coefficient

2.2 Ignoring the quadratic coefficient

The contribution of the quadratic coefficient, c_2 in the delay polynomial to the total delay is $c_2 T^2$ (where T is the time measured in ticks). Ignoring the quadratic term, c_2, in the delay polynomial will work so long as the polynomial is used for a time interval shorter than T_{max}, given by $c_2 T_{\text{max}}^2 < t_{\text{res}}$. With the above numbers, this implies that

$$t_{\text{max}} < 1.46\,\text{s}, \tag{1}$$

so that the quadratic term is not relevant for one-second intervals.

The expected maximum delay rate for terrestrial observation is $1.6 \times 10^{-6}\,\text{s/s} = 1.6 \times 10^{-6}\,\text{ticks/tick}$. Since $\log_2(1.6 \times 10^{-6} = -19.25$, the first 19 bits after the binary point in the register will be empty. To accommodate this without underflowing the register, the 32-bit delay rate coefficient is proposed to have a further 20 bits after the binary point in addition to the 8-bit fixed point proposed for the constant delay register for a total of 28 bits for the fractional part.

An important detail is that the resolution of the coefficients need not match that of the registers used internally for computing delay and phase, so mixing fixed-point numbers of different precisions is not a problem.

2.3 Shifting the linear coefficient

We now turn to estimating the errors that will result from the fixed-point representations of the constant and linear coefficients. The error (in seconds) for the constant

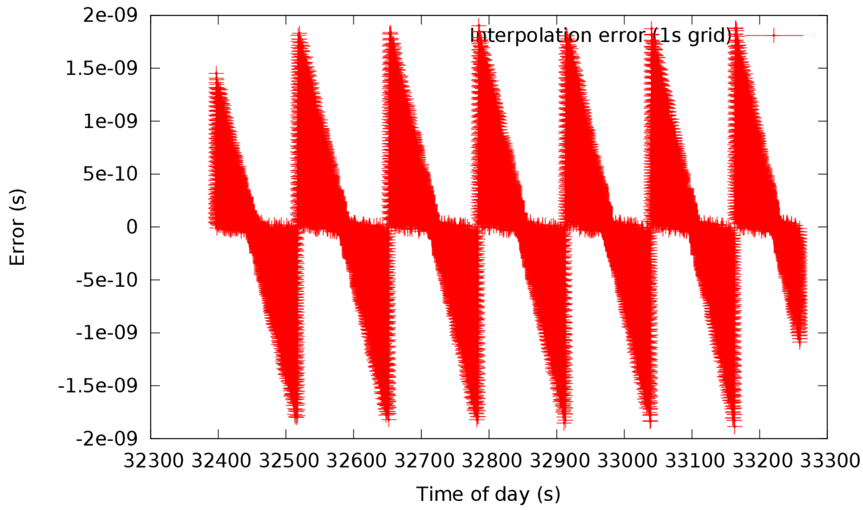

Figure 2: Delay errors with 28-bit shifted linear coefficient

coefficient is

$$\mathrm{Err}_0 = \pm\frac{0.5 \times 2^{-8}}{32 \times 10^6} = 6.1 \times 10^{-11}\,\mathrm{s} \tag{2}$$

while the error due to the linear term over an interval of $1\mathrm{s}(= 32 \times 10^6$ ticks) is given by

$$\mathrm{Err}_1 = \pm\frac{0.5 \times 2^{-28}}{32 \times 10^6} \cdot 32 \times 10^6 \approx 1.86 \times 10^{-9}\,\mathrm{s}. \tag{3}$$

To compare this with experimental values, we have evaluated the errors using the proposed model representation for the first scan of the EVN experiment EG065A. The results are shown in Figure 2.3, with a close-up in Figure 2.3 (See below for the methodology of the comparisons.)

The errors can be seen to peak at $\pm 2 \times 10^{-9}$, in good agreement with our prediction. This error is *not* less than the time resolution of 1.2×10^{-10} s (see Section 2). This suggests that we should reconsider the representation of the linear coefficient to achieve greater accuracy.

Figure 2.3 below shows the errors for a 32-bit shifted linear coefficient. The errors for this case are less than 1.5×10^{-10} s, which is close to the time resolution of the JUC's delay model.

2.4 Details of comparisons

To analyse the effects of using discrete coefficients to calculate delay and phase values we have fitted the coefficients to model delays based on a one-second interval, where we begin the scan a second early to allow the quadratic coefficients to be handled uniformly (without making a special case for the first interval).

We then evaluate the delay and phase (with a frequency 22.4 Ghz, since this is the highest ever used in practice) at 1/32 second intervals and compare with an interpolation using the known-good method of Akima splines, as used in SFXC, at the same time values.

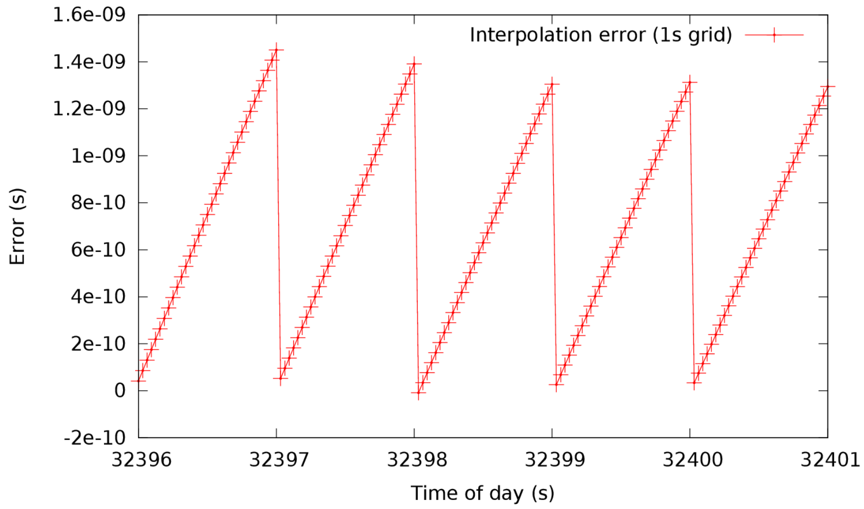

Figure 3: Delay errors with 28-bit shifted linear coefficient (detail)

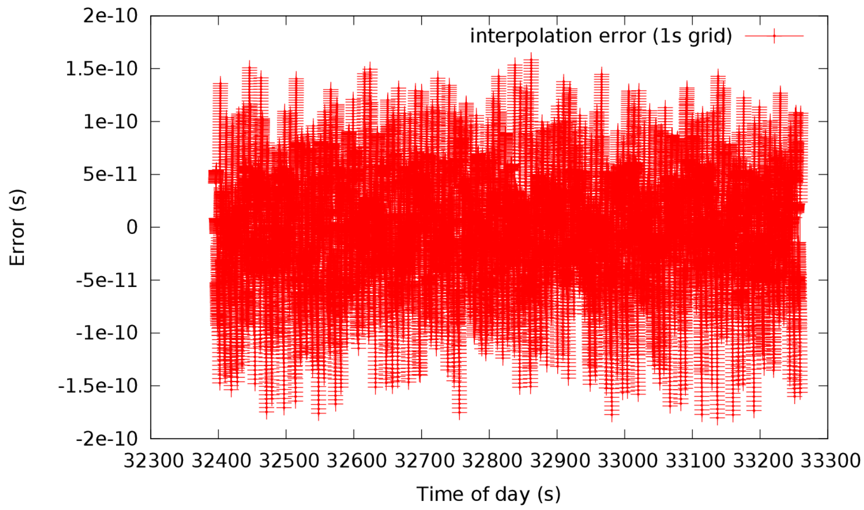

Figure 4: Delay errors with 32-bit shifted linear coefficient

2.5 Revising the proposal for delay coefficients

At least for earth-based observations, the quadratic coefficient is unnecessary. The linear term, on the other hand, needs to be shifted by at least 32-bits to preserve enough precision. Since all coefficients were originally proposed to be 32-bits long, I recommend dropping the quadratic term and expanding the constant and linear terms to 48 bits each. This allows 1 s intervals to be used rather than the proposed $1/32$ s intervals, so it is still much more parsimonious in terms of data transfer from the control system to the correlator.

3 Phase accuracy

3.1 Rationale for 48-bit registers

Sergei explained that the phase registers were sized at 48 bits to allow a correlation to run for 24 hours with a cumulative error in the phase rate of less that 0.01cycles. It is easily checked that with 48-bit registers the error in the phase rate Err $\left(\frac{d\phi}{dt}\right)$ does indeed satisfy this constraint:

$$\text{Err}\left(\frac{d\phi}{dt}\right) == 32 \times 10^6 \times (24 \times 60 \times 60) \times 2^{-48} < 0.01 \text{ cycles.} \tag{4}$$

In the Mark 3 and Mark 4 correlators the phase rate is the relevant consideration since the phase is loaded only at the beginning of the integration period; from then on it is updated only from the phase-rate register.

Generalising Equation 4 we have

$$32 \times 10^6 t_{max} 2^{-\text{nbits}} = 0.01, \tag{5}$$

so that

$$t_{max} 2^{-\text{nbits}} = 3.125 \times 10^{-10}.2^{\text{nbits}}. \tag{6}$$

Plugging the numbers into the formula above, we get the results shown in Table 3.1.

Number of bits	Maximum time
16	2.05×10^{-5} s
24	5.24×10^{-3} s
32	1.34 s
40	343 s (>3 min)
48	88000 s (>24 h)

Table 1: Maximum time for phase register bit size

3.2 A new rationale for 60-bit registers

With the proposed handling of the model in the JUC the constraint described in the previous section isn't necessary: we anticipate resetting the phase value directly every second from the constant coefficient of the delay polynomial.

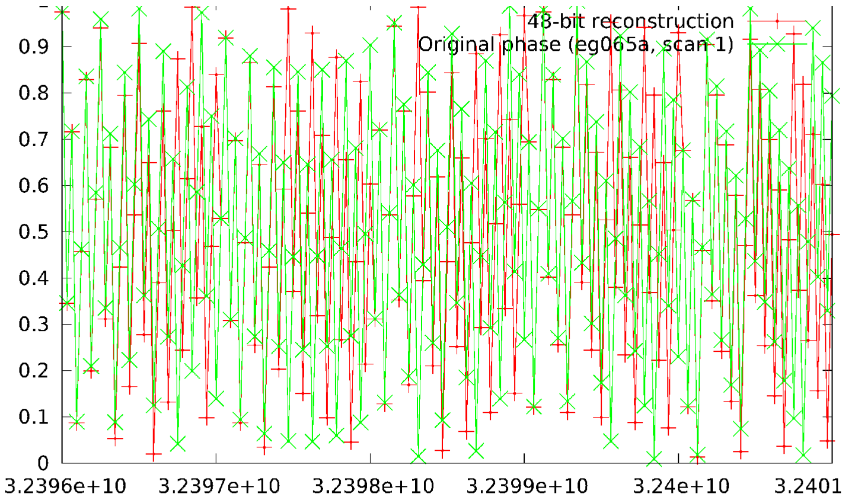

Figure 5: Phase reconstruction from 48-bit coefficients

However, this is not the only constraint. It is proposed to add the nine most significant bits of the phase value to the inputs of the poly-phase filter-bank, so our polynomial must have this level of precision over an interval of one second, or 32×10^6 ticks.

It follows that

$$\mathrm{Err}(c_2)T_{\mathrm{max}}^2 \leq 0.5 \times 2^{-9} \tag{7}$$

so that

$$\mathrm{Err}(c_2) \leq 9.54 \times 10^{-19} \tag{8}$$

which is to say that the second-order coefficient needs 60 bits.

Figure 5 shows the reconstruction of phase based on 48-bit coefficients, with the error shown in Figure 6.

Figure 7 shows the *error* for 59-bit coefficients, from which it can be seen that the accuracy in this case is acceptable over a one-second interval.

4 Space VLBI

For the space telescope Radio ASTRON delays and delay rates will be significantly higher than for purely terrestrial observations.

4.1 Delays

For Radio ASTRON delays can be up to $2\ s$ (64×10^6 ticks, requiring 26 bits of storage for the integer part). If we want to store coefficients with 8-bit fixed-point, we need at least 34 bits. In particular, 32-bit coefficients aren't enough. The next "standard" size up is 48 bits, so we now check that's enough for delay rates.

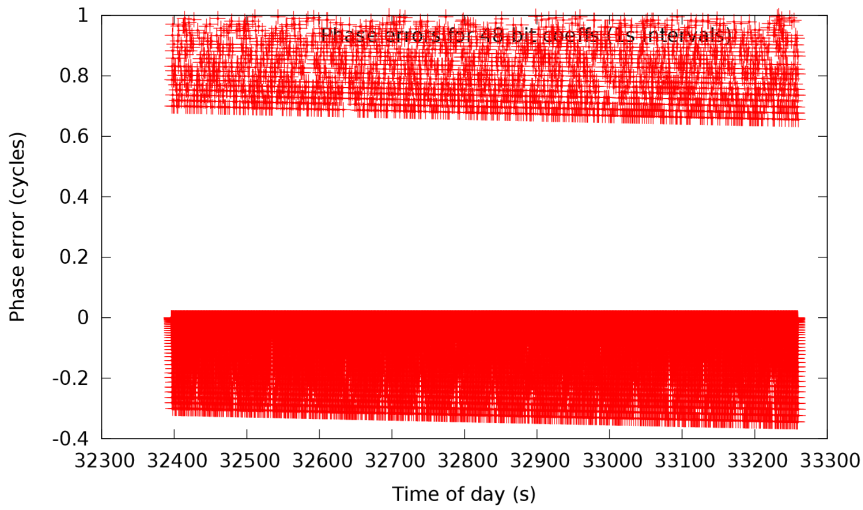

Figure 6: Phase error for 48-bit coefficients

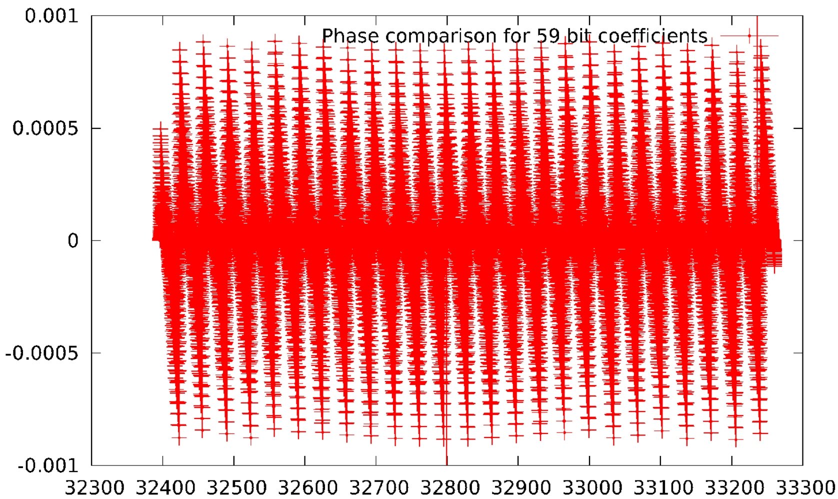

Figure 7: Phase error for 48-bit coefficients

4.2 Delay rates

For Radio ASTRON delay rates can be up to $50\,\mu s/\mathrm{s}$. This fits comfortably into a 32-bit coefficient, even when coefficients are shifted up by 32 bits – in this case we can effectively represent any value smaller than unity. So 48-bit coefficients aren't really necessary, but given that they are only transmitted once a second instead of 32 times, the additional overhead doesn't seem too extravagant.

4.3 Delay acceleration

Delay acceleration can safely be ignored in the terrestrial case, even over one-second intervals. This may not be the case for Radio ASTRON. (I don't have a number for the worst expected case.)

If we really want to be space-capable, we should probably include a 48-bit acceleration coefficient, and have a flag somewhere to ignore it for terrestrial correlations.

4.4 Phase

Phase goes from zero to one (in units of cycles) whether observations are terrestrial or involve space telescopes, so the phase coefficients need not be reconsidered.

5 Conclusions and Recommendations

We conclude that:

- 32-bits are not quite enough to calculate delays for one-second intervals, but 48-bit linear polynomials are more than enough for earth-based VLBI. An open question is whether we would need a quadratic term for space VLBI; we could always include it and have a flag to ignore it.

- 60-bit quadratic polynomials are good enough for phase on one-second intervals; smaller coefficients won't do. So we should make the coefficients a round 64 bits and pad the least-significant with zeroes to taste.